Q1

1a

The result required is in completed square form.
As the coefficient of x^2 is not 1, begin by factorising the number 2 from the first two terms of the expression.

$$2x^2 + 16x + 35 = 2(x^2 + 8x) + 35$$

[1]

Complete the square on the expression in the brackets.
Set up a squared bracket with two terms; x and half of the coefficient of the x term - in this case 4.
Considering that $(x+4)^2 = x^2 + 8x + 16$, we need to subtract 16.

$$2(x^2 + 8x) + 35 = 2[(x + 4)^2 - 16] + 25$$

Multiply out the 2.

$$2x^2 + 16x + 35 = 2[(x + 4)^2 - 16] + 35$$
$$= 2(x + 4)^2 - 32 + 35$$

[1]

$$= 2(x + 4)^2 + 3$$

$\mathbf{a = 2, b = 4, c = 3}$ [1]

1b

The turning point of $a(x + b)^2 + c$ is at the point $(-b, c)$.

$$b = 4, c = 3$$

$\mathbf{(-4, 3)}$ [1]

Q2

2a

Completing the square is easiest when dealing with a single, positive x^2 term, so take a factor of -3.
To avoid fractions, take this factor out of the x^2 and x terms only.

$$7 + 12x - 3x^2 = 7 - 3(x^2 - 4x)$$

[1]

Complete the square for the part in brackets.

$$7 - 3(x^2 - 4x) = 7 - 3[(x - 2)^2 - 4]$$

[1]

Expand and rearrange into the required form.

$$7 - 3[(x - 2)^2 - 4] = 7 - 3(x - 2)^2 + 12$$
$$= 19 - 3(x - 2)^2$$

$\mathbf{\therefore 7 + 12x - 3x^2 = 19 - 3(x - 2)^2}$ [1]

i.e. $a = 19,\ b = -3,\ c = -2.$

2b

The maximum point is found by completing the square.

$$7 + 12x - 3x^2 - 12x + 7 = 19 - 3(x - 2)^2$$

The maximum point of $a + b(x + c)^2$ is $(-c, a)$.

$$\therefore \text{Max. point is } (2, 19)$$

You can also deduce this logically - the maximum value will be when $(x - 2) = 0$ as this will be the least amount possible to subtract from 19. $x - 2 = 0$ when $x = 2$ and of course $19 - 0 = 19$.

The coordinates of A, the maximum point are $(2, 19)$ [1]

Q3

3

Completing the square is easiest when dealing with a single, positive x^2 term, so take a factor of -2. To avoid fractions, take this factor out of the x^2 and x terms only.

$$7-12x-2x^2=7-2(x^2+6x)$$

[1]

Complete the square for the part in brackets.

$$7-2(x^2+6x)=7-2[(x+3)^2-9]$$

[1]

Expand and rearrange into the required form.

$$7-2[(x+3)^2-9]=7-2(x+3)^2+18$$
$$=25-2(x+3)^2$$

$\therefore 7-12x-2x^2=25-2(x+3)^2$ [1]

i.e. $a=25,\ b=-2,\ c=3.$

Q4

4

Completing the square is easiest when dealing with a single, positive x^2 term, so take a factor of -1. This is only necessary for the x^2 and x terms so leave 5 as it is.

$$5+6x-x^2=5-(x^2-6x)$$

Complete the square for the part in brackets.

$$5-(x^2-6x)=5-[(x-3)^2-9]$$

[1]

Expand and rearrange into the required form.

$$5-[(x-3)^2-9]=5-(x-3)^2+9$$
$$=14-(x-3)^2$$

$\therefore f(x)=5+6x-x^2=14-(x-3)^2$ [1]

i.e. $p=14,\ q=3.$

Q5

5

Completing the square is easiest when dealing with a single, positive x^2 term, so take a factor of -2. To avoid fractions, take this factor out of the x^2 and x terms only.

$$5+12x-2x^2=5-2(x^2-6x)$$

[1]

Complete the square for the part in brackets.

$$5-2(x^2-6x)=5-2[(x-3)^2-9]$$

[1]

Expand and rearrange into the required form.

$$5-2[(x-3)^2-9]=5-2(x-3)^2+18$$
$$=23-2(x-3)^2$$

[1]

$\therefore 5+12x-2x^2=23-2(x-3)^2$ [1]

i.e. $a=23,\ b=-2,\ c=-3.$

Q6

6

Completing the square is easiest when dealing with a single, positive x^2 term, so take a factor of -1.
This is only necessary for the x^2 and x terms so leave 7 as it is.

$$7-4x-x^2=7-(x^2+4x)$$

Complete the square for the part in brackets.

$$7-(x^2+4x)=7-[(x+2)^2-4]$$

[1]

Expand and rearrange into the required form.

$$7-[(x+2)^2-4]=7-(x+2)^2+4$$
$$=11-(x+2)^2$$

$\therefore 7-4x-x^2=11-(x+2)^2$ [1]

i.e. $p=11,\ q=2$.

Q7

Factorise the 2 out of the first two terms

$$2[x^2-3x]+5$$

[1]

Complete the square on the expression inside the square brackets (by writing it as $(x+p)^2 - p^2$ where p is half of -3)

$$2\left[\left(x-\frac{3}{2}\right)^2-\left(-\frac{3}{2}\right)^2\right]+5$$
$$=2\left[\left(x-\frac{3}{2}\right)^2-\frac{9}{4}\right]+5$$

correct inside square-shaped brackets [1]

Expand the square-shaped brackets

$$2\left(x-\frac{3}{2}\right)^2-\frac{9}{2}+5$$

Simplify $-\frac{9}{2}+5$ into one number

$$2\left(x-\frac{3}{2}\right)^2+\frac{1}{2}$$

This is in the form $a(x-b)^2+c$
Read off a, b and c (b will be positive, as the negative sign is already given in $x-b$)

$a=2,\ b=\frac{3}{2},\ c=\frac{1}{2}$ [1]

$a=2$, $b=1.5$ and $c=0.5$ are also accepted

Q8

8

The formula for completing the square for a quadratic in the form x^2+bx+c is $\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c$.

Complete the square.

$$(x+2)^2-4-3$$

Correct expression in the brackets [1]
Correct values outside of the brackets (may be simplified) [1]

Simplify.

$$(x+2)^2-7$$

[1]

The turning point of $(x+a)^2-b$ is at the point $(-a,b)$

$(-2,-7)$ [1]